Notes: Sequences (Section 4-2 INT 3)

An explicit formula for a sequence gives the value of any term $a_n$ in terms of $n$ (position of the term).

**Example 1:**
Write an explicit formula for each sequence

a. 11, 17, 23, 29,...  

\[ a_n = 11 + 6(n-1) \]

b. 3, 9, 27, 81,...  

\[ a_n = 3^n \]

c. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5},...$  

\[ a_n = \frac{n}{n+1} \]

**Example 2:**
Write the first three terms and the 12th term of each sequence.

a. $a_n = 3n - 5$  

\[
\begin{align*}
a_1 &= 3(1) - 5 = -2 \\
a_2 &= 3(2) - 5 = 1 \\
a_3 &= 3(3) - 5 = 4 \\
a_{12} &= 3(12) - 5 = 31
\end{align*}
\]

b. $b_n = 2^{1-n}$  

\[
\begin{align*}
b_1 &= 2^{1-1} = 2^0 = 1 \\
b_2 &= 2^{1-2} = 2^{-1} = \frac{1}{2} \\
b_3 &= 2^{1-3} = 2^{-2} = \frac{1}{4} \\
b_{12} &= 2^{1-12} = 2^{-11} = \frac{1}{2048}
\end{align*}
\]

Notes, Using Recursive Formulas

An explicit formula uses the position of a term to give the value of that term in the sequence.

A recursive formula uses the previous terms to get to the next term. Every recursive formula has at least two parts:

1. Starting value for $a_1$ (first term)
2. Recursion equation (rule using previous terms)

**Example 3:** Explicit vs Recursive

2, 4, 6, 8, 10, ...  

Explicitly would be:

\[ a_n = 2n \]

Recursively:

\[
\begin{align*}
a_1 &= 2 \\
a_{n+1} &= a_n + 2
\end{align*}
\]
Example 4: \[ 3, 9, 27, 81, \ldots \]

Explicitly would be: \[ a_n = 3^n \]

Recursively:

\[
\begin{align*}
  a_1 &= 3 \\
  a_n &= 3 \cdot (a_{n-1})
\end{align*}
\]

Example 5: Write a recursive formula for the sequence 1, 2, 6, 24, \ldots

\[
\begin{align*}
  a_1 &= 1 \\
  a_n &= n \cdot (a_{n-1})
\end{align*}
\]

Write an explicit formula for the same sequence:

\[ a_n = n! \]

Example 6: Write the first four terms of each sequence.

\[
\begin{align*}
  a_1 &= -1 \\
  a_n &= 3a_{n-1} + 4
\end{align*}
\]

\[
\begin{align*}
  a_2 &= 3(a_1) + 4 = 3(-1) + 4 = 1 \\
  a_3 &= 3(a_2) + 4 = 3(1) + 4 = 7 \\
  a_4 &= 3(a_3) + 4 = 3(7) + 4 = 25
\end{align*}
\]

\[
\begin{align*}
  b_1 &= 5 \\
  b_2 &= 1 \\
  b_n &= b_{n-1} - b_{n-2}
\end{align*}
\]

\[
\begin{align*}
  b_3 &= b_{3-1} - b_{3-2} = b_2 - b_1 = 1 - 5 = -4 \\
  b_4 &= b_{3} - b_2 = -4 - 1 = -5
\end{align*}
\]
Notes: Arithmetic and Geometric Sequences

A sequence in which the difference between any term and the term before is a constant is an arithmetic sequence.

EX: \[2, 4, 6, 8, 10, \ldots\]
\[d = \frac{any\ term - previous\ term}{previous\ term} = a_n - (a_{n-1})\]

A sequence in which the ratio of any term to the term before it is a constant is a geometric sequence.

EX: \[2, 4, 8, 16, \ldots\]
\[r = \frac{any\ term}{previous\ term} = \frac{a_n}{a_{n-1}}\]

Example 7: Tell whether each sequence is arithmetic, geometric, or neither.

A. 27, 9, 3, 1, \ldots geometric \[r = \frac{1}{3}\]

B. 1, 3, 7, 15, \ldots neither - no common difference
\[\text{no constant ratio}\]

C. 12, 9.5, 7, 4.5, \ldots arithmetic \[d = -2.5\]
General Formulas for Arithmetic Sequences

Explicit Formula

\[ a_n = a_1 + (n-1)d \]

Recursive Formula

\[ a_1 = \text{value of the first term} \]
\[ a_n = a_{n-1} + d \]

Example

3, 5, 7, 9, \ldots

\[ a_n = 3 + (n-1) \cdot 2 \]
\[ a_n = 2n + 1 \] (Explicit)

\[ a_1 = 3 \]
\[ a_n = a_{n-1} + 2 \] (Recursive)

We can use an explicit formula to find the number of terms in a finite sequence that is arithmetic or geometric.

**Example 8**: Tina is knitting a sweater with a repeating triangle pattern. The pattern repeat for each triangle is to knit 33 stitches, purl 29 stitches, knit 25 stitches, purl 21 stitches, and so on, ending with 1 knit stitch. How many rows are there in each triangle?

\[ a_n = a_1 + (n-1)d \]
\[ 1 = 33 + (n-1)(-4) \]
\[ -32 = (n-1)(-4) \]
\[ 8 = n-1 \]
\[ q = n \]

\[ n = 9 \text{ rows} \]
General Formulas for Geometric Sequences

Explicit Formula

\[ a_n = a_1 \cdot r^{n-1} \]

Recursive Formula

\[ a_1 = \text{value of the first term} \]
\[ a_n = (a_{n-1})r \]

**Example 9**: Find the ninth term of the geometric sequence

-2, 6, -18, 54, . . .

\[ a_1 = -2 \]
\[ r = -3 \]

\[ a_9 = a_1 \cdot r^{n-1} \]
\[ a_9 = -2 \cdot (-3)^8 \]
\[ a_9 = -2 \cdot 6561 \]
\[ a_9 = -13122 \]