Notes: Sequences (Section 4-2 INT 3)

An **explicit formula** for a sequence gives the value of any term a_n in terms of n (position of the term)

Example 1:
Write an explicit formula for each sequence

a. 11, 17, 23, 29,...

1 2 3 4

$a_1 = 11$
$a_2 = 11 + 6(1)$
$a_3 = 11 + 6(2)$
$a_n = 11 + 6(n-1) = 6n+5$

b. 3, 9, 27, 81,...

1 2 3 4

$a_1 = 3^1$
$a_2 = 3^2$
$a_3 = 3^3$
$\boxed{a_n = 3^n}$

c. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

1 2 3 4

$a_n = \frac{n}{n+1}$

Example 2:
Write the first three terms and the 12th term of each sequence. a_1, a_2, a_3, a_{12}

a. $a_n = 3n - 5$

$a_1 = 3(1) - 5 = -2$
$a_2 = 3(2) - 5 = 1$
$a_3 = 3(3) - 5 = 4$
$a_{12} = 3(12) - 5 = 31$

b. $b_n = 2^{1-n}$

$b_1 = 2^{1-1} = 2^0 = 1$
$b_2 = 2^{1-2} = 2^{-1} = \frac{1}{2}$
$b_3 = 2^{1-3} = 2^{-2} = \frac{1}{4}$
$b_{12} = 2^{1-12} = 2^{-11} = \frac{1}{2048}$

Notes, Using Recursive Formulas

An **explicit formula** uses the position of a term to give the value of that term in the sequence

A **recursive formula** uses the previous terms to get to the next term. Every recursive formula has at least two parts:

1. Starting value for a_1 (first term)
2. recursion equation (rule using previous terms) a_{n-1}

Example 3: Explicit -vs- Recursive

1 2 3 4 5

2, 4, 6, 8, 10, . . .

+2 +2 +2

Explicitly would be: $a_n = 2n$

Recursively: $a_1 = 2$
$a_n = a_{n-1} + 2$

SAME

$a_1 = 2$
$a_{n+1} = a_n + 2$

Example 4: 1 2 3 4

3, 9, 27, 81, . . .

$\cdot 3 \quad \cdot 3 \quad \cdot 3$

Explicitly would be: $a_n = 3^n$

Recursively: $a_1 = 3$

$a_n = 3 \cdot (a_{n-1})$

Example 5: Write a recursive formula for the sequence

1 2 3 4

1, 2, 6, 24, . . .

$\cdot 2 \quad \cdot 3 \quad \cdot 4$

$a_1 = 1$

$a_n = n \cdot (a_{n-1})$

Write an explicit formula for the same sequence: $a_n = n!$

factorial

$a_1 = 1$

$a_2 = 2 \cdot 1$

$a_3 = 3 \cdot 2 \cdot 1$

$a_4 = 4 \cdot 3 \cdot 2 \cdot 1$

Example 6: Write the first four terms of each sequence.

$a_1 = -1$

$a_n = 3a_{n-1} + 4$

$a_2 = 3(a_{2-1}) + 4$

$= 3(a_1) + 4 = 3(-1) + 4$

$a_2 = 1$

$a_3 = 3(a_2) + 4 = 7$

$a_4 = 3(a_3) + 4 = 25$

$\boxed{\begin{array}{l} b_1 = 5 \\ b_2 = 1 \end{array}}$

$b_n = b_{n-1} - b_{n-2}$

$b_3 = b_{3-1} - b_{3-2}$

$= b_2 - b_1 = 1 - 5 = -4$

$b_4 = b_3 - b_2 = -4 - 1 = -5$

Notes: Arithmetic and Geometric Sequences

A sequence in which the difference between any term and the term before is a constant is an **arithmetic sequence**.

common difference

$d =$ any term $-$ previous term $= a_n - (a_{n-1})$

EX: 2, 4, 6, 8, 10, . . .
+2 +2 +2

A sequence in which the ratio of any term to the term before it is a constant is a **geometric sequence**. constant ratio: r

EX: 2, 4, 8, 16, . . .
·2 ·2 ·2

$r = \dfrac{\text{any term}}{\text{previous term}} = \dfrac{a_n}{a_{n-1}}$

Example 7: Tell whether each sequence is arithmetic, geometric, or neither.

A. 27, 9, 3, 1, . . .
÷3 ÷3 ÷3
geometric $r = \frac{1}{3}$

B. 1, 3, 7, 15, . . .
+2 +4 +8
neither - no common difference & no constant ratio

C. 12, 9.5, 7, 4.5, . . .
−2.5 −2.5 −2.5
arithmetic $d = -2.5$

General Formulas for Arithmetic Sequences

Explicit Formula

$a_n = a_1 + (n-1)d$

common difference

first term

position

Example

3, 5, 7, 9, . . .

+2 +2 +2

$a_n = 3 + (n-1) \cdot 2$

$\boxed{a_n = 2n + 1}$ Explicit

Recursive Formula

$a_1 =$ value of the first term

$a_n = a_{n-1} + d$

$$\boxed{\begin{aligned} a_1 &= 3 \\ a_n &= a_{n-1} + 2 \end{aligned}}$$

Recursive

We can use an explicit formula to find the number of terms in a finite sequence that is arithmetic or geometric

Example 8: Tina is knitting a sweater with a repeating triangle pattern. The pattern repeat for each triangle is to knit 33 stitches, purl 29 stitches, knit 25 stitches, purl 21 stitches, and so on, ending with 1 knit stitch. How many rows are there in each triangle?

n?

33, 29, 25, 21, . . . , 1

−4 −4 −4 . . .

$a_n = a_1 + (n-1)d$

$1 = 33 + (n-1)(-4)$

$-32 = (n-1)(-4)$

$8 = n - 1$

$9 = n$

n = 9 rows

General Formulas for Geometric Sequences

Explicit Formula

$$a_n = a_1 \cdot r^{n-1}$$

constant ratio

first term

Example

3, 6, 12, 24, . . . $r=2$

·2 ·2 ·2

$$\boxed{a_n = 3 \cdot (2)^{n-1}}$$

Explicit

Recursive Formula

$a_1 =$ value of the first term

$a_n = (a_{n-1})r$ ← constant ratio

previous term

$$\boxed{\begin{aligned} a_1 &= 3 \\ a_n &= (a_{n-1}) \cdot 2 \end{aligned}}$$

recursive

Example 9: Find the ninth term of the geometric sequence

-2, 6, -18, 54, . . .

a_1 $r=-3$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_9 = -2 \cdot (-3)^{9-1}$$

$$a_9 = -2(-3)^8$$

$$\boxed{a_9 = -13122}$$